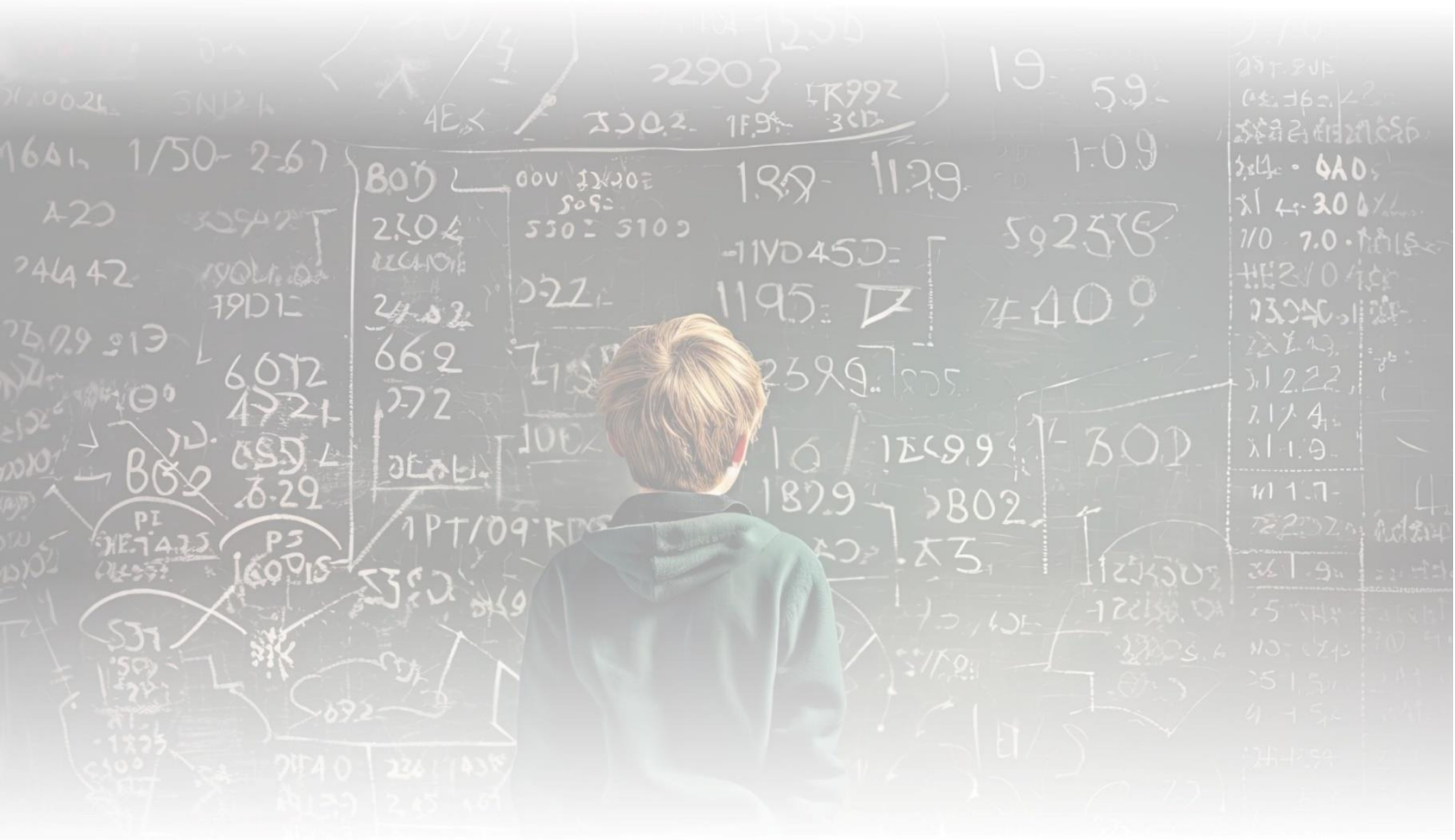
Smart, Structured & Syllabus-Aligned Content

RICE EDUCATION



MATHS

WBCS

LCM & HCF

(REFERENCE MATERIAL)

LCM & HCF

(Least Common Multiple & Highest Common Factor)

LCM and HCF are two fundamental properties of integers. HCF is the largest number that can completely divide all given numbers, whereas LCM is the smallest number that is completely divisible by all given numbers.

1. Highest Common Factor (HCF / GCD)

The HCF of two or more numbers is the greatest number that divides each number without leaving any remainder.

Example (for 12 and 18):

Factors of 12 = 1, 2, 3, 4, 6, 12

Factors of 18 = 1, 2, 3, 6, 9, 18

Common factors = 1, 2, 3, 6

HCF = 6

2. Least Common Multiple (LCM)

The LCM of two or more numbers is the smallest non-zero number that is a multiple of each of the given numbers.

Example (for 12 and 18):

Multiples of 12 = 12, 24, 36, 48, 60, 72, ...

Multiples of 18 = 18, 36, 54, 72, 90, ...

Common multiples = 36, 72, 108, ...

LCM = 36

Methods for Finding HCF and LCM:

1. **Prime Factorization Method:** First, express each number in terms of its prime factors.
   HCF: The product of the lowest powers of common prime factors.
   LCM: The product of the highest powers of all prime factors involved.

Characteristics and Essential Formulas

Formula 1 (Product of two numbers): For any two numbers a and b, $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$

(Note: This rule applies only to two numbers, not for three or more numbers.)

Formula 2 (HCF and LCM of fractions):

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

Formula 3 (Co-prime Ratio Concept): If the HCF of two numbers a and b is H, then the numbers can be written as $a = H \times x$ and $b = H \times y$ (where x and y are co-prime, meaning $\text{HCF}(x,y) = 1$). From this concept, $\text{LCM} = H \times x \times y = \text{HCF} \times x \times y$.

Problem-Solving Formulas

Type 1: HCF Problems (Group Division):

Find the greatest number that divides A, B, C leaving no remainder: $\text{HCF}(A,B,C)$.

Find the greatest number that divides A, B, C leaving remainders r_1, r_2, r_3 respectively: $\text{HCF}(A-r_1, B-r_2, C-r_3)$.

Find the greatest number that divides A, B, C leaving the same remainder in each case: $\text{HCF}(|A-B|, |B-C|, |C-A|)$.

Type 2: LCM Problems (Cycles / Minimum Quantity):

Find the smallest number that is completely divisible by A, B, C: $\text{LCM}(A,B,C)$.

Find the smallest number which when divided by A, B, C leaves a remainder k in each case: $\text{LCM}(A,B,C) + k$.

When bells toll together and then ring at intervals of T_1, T_2, T_3, or traffic lights change: Next simultaneous occurrence time = $\text{LCM}(T_1, T_2, T_3)$.

Important Notes

HCF × LCM = Product of two numbers.

For any set of numbers, the HCF is always a factor of the LCM.

For simultaneous events (such as bells ringing, traffic lights changing, running on a circular track), you need to find the LCM of the separate time intervals.

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PREVIOUS YEAR QUESTION

1. The HCF and LCM of two numbers are 12 and 180 respectively. If one number is 36, what is the other number?
(A) 48
(B) 60
(C) 72
(D) 90
Answer: (B) 60
Solution: According to the formula, 1st number × 2nd number = HCF × LCM.
$36 \times$ 2nd number $= 12 \times 180 \Longrightarrow$ 2nd number $= \frac{12\times180}{36} = 60.$

2. The HCF of two numbers is 8. Which of the following can never be their LCM?
(A) 24
(B) 48
(C) 56
(D) 60
Answer: (D) 60
Solution: Let us divide the options by the HCF (8):
$24 \div 8 = 3$ (Divisible)
$48 \div 8 = 6$ (Divisible)
$56 \div 8 = 7$ (Divisible)
$60 \div 8 = 7.5$ (Not completely divisible, so it cannot be the LCM)

3. What is the greatest number which, when it divides 1356, 1868, and 2764, leaves a remainder of 12 in each case?
(A) 64
(B) 124
(C) 156
(D) 260
Answer: (C) 156
Solution: Subtract 12 from each number:
$1356 - 12 = 1344, 1868 - 12 = 1856, 2764 - 12 = 2752.$
Now we find the HCF of the mutual differences of these numbers:
$1856 - 1344 = 512, 2752 - 1856 = 896.$
$\text{HCF}(512,896) = 156.$

4. Three bells ring at intervals of 9, 12, and 15 minutes respectively. If they ring together now, after how many hours will they ring together again?
(A) 3 hours
(B) 6 hours
(C) 9 hours
(D) 12 hours
Answer: (A) 3 hours
Solution: Find the LCM of 9, 12, and 15:
$9 = 3^2, 12 = 2^2 \times 3, 15 = 3 \times 5.$
$\text{LCM} = 2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$ minutes.

Converting minutes into hours: $180 \div 60 = 3$ hours.

5. Which is the smallest square number that is completely divisible by 3, 4, 5, 6, and 8?
(A) 900
(B) 1200
(C) 2500
(D) 3600
Answer: (D) 3600
Solution: LCM$(3,4,5,6,8) = 120$.
Prime factorization of 120 = $2^3 \times 3 \times 5$.
To make it a perfect square, multiply by the unmatched prime factors (2,3,5), which gives $2 \times 3 \times 5 = 30$.
Required square number = $120 \times 30 = 3600$.

6. The ratio of two numbers is 3:4. If their HCF is 4, what is their LCM?
(A) 12
(B) 16
(C) 24
(D) 48
Answer: (D) 48
Solution: Let the numbers be $3x$ and $4x$, where x = HCF $= 4$.
LCM = HCF × Product of ratio terms $= 4 \times 3 \times 4 = 48$.

7. Find the HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$, and $\frac{10}{27}$.
(A) $\frac{2}{81}$
(B) $\frac{16}{3}$
(C) $\frac{2}{3}$
(D) $\frac{80}{81}$
Answer: (A) $\frac{2}{81}$
Solution: HCF of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$.
HCF$(2,8,16,10) = 2$.
LCM$(3,9,81,27) = 81$.
Required HCF $= \frac{2}{81}$.

8. Find the smallest number which when divided by 12, 16, 18, and 21 leaves a remainder of 7 in each case.
(A) 1008
(B) 1015
(C) 1022
(D) 1032
Answer: (B) 1015
Solution: LCM$(12,16,18,21) = 1008$.
Add the remainder: Required number = $1008 + 7 = 1015$.

9. What is the largest 4-digit number that is completely divisible by 12, 15, 18, and 27?
(A) 9720
(B) 9930
(C) 9860
(D) 9960
Answer: (A) 9720
Solution: LCM$(12,15,18,27) = 540$.
The largest 4-digit number is 9999.
Dividing 9999 by 540 leaves a remainder of 279.
Required number = $9999 - 279 = 9720$.

10. The sum of two numbers is 384 and their HCF is 48. How many such pairs of numbers are possible?
(A) 1
(B) 2
(C) 3
(D) 4
Answer: (B) 2
Solution: Let the numbers be $48a$ and $48b$ (where a and b are co-prime).
$48(a+b) = 384 \Rightarrow a+b = 8$.
Co-prime pairs whose sum is 8 are: $(1,7)$ and $(3,5)$. Thus, only 2 pairs are possible.

11. The LCM of two numbers is 12 times their HCF. The sum of HCF and LCM is 403. If one number is 93, what is the other number?
(A) 124
(B) 134
(C) 128
(D) 138
Answer: (A) 124
Solution: $\text{LCM} + \text{HCF} = 12 \times \text{HCF} + \text{HCF} = 13 \times \text{HCF} = 403 \Rightarrow \text{HCF} = 31$.
$\text{LCM} = 12 \times 31 = 372$.
Other number $= \frac{\text{HCF} \times \text{LCM}}{\text{Given number}} = \frac{31 \times 372}{93} = 124$.

12. Traffic lights at three different road crossings change every 48 seconds, 72 seconds, and 108 seconds respectively. If they change together at 8:20:00 AM, when will they change together again?
(A) 8:27:12 AM
(B) 8:25:20 AM
(C) 8:24:12 AM
(D) 8:28:00 AM
Answer: (A) 8:27:12 AM
Solution: $\text{LCM}(48,72,108) = 432$ seconds = 7 minutes 12 seconds.
Exact time = 8:20:00 + 0:07:12 = 8:27:12.

13. Find the smallest number which when divided by 20, 25, 35, and 40 leaves remainders of 14, 19, 29, and 34 respectively?
(A) 1394
(B) 1404
(C) 1396
(D) 1400
Answer: (A) 1394
Solution: Common difference between divisor and remainder = $20 - 14 = 6$. LCM of 20, 25, 35, and 40 = 1400. Required number = $1400 - 6 = 1394$.

14. What is the LCM of $\frac{1}{3}, \frac{2}{9}, \frac{5}{6}$, and $\frac{4}{27}$?
(A) $\frac{1}{54}$
(B) $\frac{10}{3}$
(C) $\frac{20}{3}$
(D) $\frac{20}{27}$
Answer: (C) $\frac{20}{3}$
Solution: LCM of fractions $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}} = \frac{\text{LCM}(1,2,5,4)}{\text{HCF}(3,9,6,27)} = \frac{20}{3}$.

15. What is the greatest number that divides 62, 132, and 237 leaving the same remainder in each case?
(A) 15
(B) 25
(C) 35
(D) 45

Answer: (C) 35
Solution: Take the HCF of the mutual differences: $|132-62|=70$, $|237-132|=105$, $|237-62|=175$.
$\mathrm{HCF}(70,105,175)=35$.

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